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X-ray binary systems and nonextensivity

Marcelo A. Moret [a,b,*], Valter de Senna [a], Gilney F. Zebende [a,b], Pablo Vaveliuk [a]

[a] *Programa de Modelagem Computational - SENAI - CIMATEC 41650-010 Salvador, Bahia, Brazil*

[b] *Departamento de Física, Universidade Estadual de Feira de Santana, CEP 44031-460, Feira de Santana, Bahia, Brazil*

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ABSTRACT

We study the x-ray intensity variations obtained from the time series of 155 light curves of x-ray binary systems collected by the instrument All Sky Monitor on board the satellite Rossi X-Ray Timing Explorer. These intensity distributions are adequately fitted by q-Gaussian distributions which maximize the Tsallis entropy and in turn satisfy a nonlinear Fokker–Planck equation, indicating their nonextensive and nonequilibrium behavior. From the values of the entropic index q obtained, we give a physical interpretation of the dynamics in x-ray binary systems based on the kinetic foundation of generalized thermostatistics. The present findings indicate that the binary systems display a nonextensive and turbulent behavior.

In recent years, the generalized thermostatistical formalism (GTS) proposed by Tsallis [1] has received increasing attention due its success in the description of certain phenomena exhibiting atypical thermodynamical features. For example, the Tsallis formalism has been applied to protein folding [2], global optimization [3] and many other cases [4]. But it was with regard to systems with long-range interactions that GTS was consolidated. In fact, the first application of the Tsallis ideas to a concrete physical problem was done in analyzing a type of astrophysical system with long-range interactions called the polytropic stellar distributions [5]. Within the conventional Boltzmann–Gibbs (BG) thermostatistics, the entropy additivity law, valid for extensive systems, is not appropriate for those subjected to long-range interactions as produced by gravitational forces. On the other hand, GTS covers this class of systems since it postulates a nonextensive (nonadditive) entropy S_q such that $S_q(A+B) = S_q(A) + S_q(B) + (1-q)S_q(A)S_q(B)$, where A and B are two independent systems in the sense that $P(x, x')_{A+B} = P(x)_A P(x')_B$. We recall that $P(x)$ is the probability density distribution of the macroscopic variable x that characterizes the system. In this context, the so called entropic index q is a measure of the degree of nonextensivity. In the BG statistics, the entropy $S = -k\int P(x)\ln P(x)\mathrm{d}x$ gives rise to exponential probability density distributions, $P(x) \propto \exp(-x)$. Yet, within GTS formalism, the maximization of the q-entropy

$$S_q = k\left(1 - \int [P(x)]^q \mathrm{d}x\right) \bigg/ (q-1), \tag{1}$$

produces power-law distributions called q-exponential distributions, defined by

$$P_q(u) \propto \mathrm{e}_q^u \equiv [1 + (1-q)\,u]^{1/(1-q)}\,, \tag{2}$$

if $1 + (1-q)\,u \geq 0$ and $\mathrm{e}_q^u = 0$ otherwise. In the limit $q \to 1$ the usual BG entropy is recovered, $S_1 \equiv S_{BG}$, and the q-exponential distribution converges to the usual exponential distribution.

* Corresponding author at: Programa de Modelagem Computational - SENAI - CIMATEC 41650-010 Salvador, Bahia, Brazil. Tel.: +55 71 82272352.
E-mail address: mamoret@gmail.com (M.A. Moret).

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The long-range gravitational forces form unique structures in the Universe called *self-gravitating systems* (SGS). Almost all of the relevant structures such as stars, binary systems, galaxies, clusters or super-clusters of galaxies, among others, belong to this category. The nonextensivity of SGS is reflected in their long-range interactions. Moreover, there is another problem with the application of the BG thermostatistics to SGS: these structures have no absolute thermal equilibrium state, i.e. they are not isothermal, displaying incomplete equilibrium states characterized by time scaling properties. In fact, SGS never stop their evolution toward the absolute thermal equilibrium. Therefore, GTS seems appropriate for studying SGS since the q-exponential distributions [Eq. (2)] arising from the maximization of the q-entropy [Eq. (1)] are also solutions of the nonlinear Fokker–Planck equation [6] used to study systems evolving to thermal equilibrium. We remark that the usefulness of the Tsallis approach is also supported by the wide utilization in diverse astrophysical problems [7].

However, some recent astrophysical results related to SGS are not sufficiently forceful to be explained via GTS. For example, negative values of q were obtained by Nakamichi et al. [8] for large-scale galaxy distributions in contrast to a positive value obtained by Lavagno et al. [9] studying velocity distributions of galaxy clusters. In addition, Lima et al. [10] show that the generalized Boltzmann equation verifies the H_q theorem only if $q > 0$. Otherwise, the entropy would decrease with time, violating the second law of thermodynamics. On the other hand, the hypotheses of Wuensche et al. [11] in applying GTS to nonlinear galaxy clustering in the Universe are very wide and hard to test due to the lack of experimental data. Finally, Bernui et al. [12] obtained a q-value very close to unity in analyzing temperature fluctuations of the cosmic microwave background radiation, raising doubts as to whether the weak nonextensive behavior of such a system is not really due to instrumental noise.

The above mentioned results lead to a strong link between astrophysical systems with long-range interactions and the generalized thermostatistics. Then, the nonextensive and incomplete equilibrium nature of the most extreme SGS, i.e. x-ray binary systems (XRBS) which contain black holes or neutron stars, must be clearly seen. To accomplish this objective, we study the x-ray light curve intensity variations of 155 XRBS, collected by the instrument All Sky Monitor on board the satellite Rossi X-Ray Timing Explorer (RXTE-ASM) whose public data are available at Ref. [13]. We show that these extreme gravitating systems can be consistently interpreted in terms of GTS. In addition, from the values obtained for the nonextensive q parameter characterizing each XRBS group, we propose a physical interpretation of such systems.

From the 155 astrophysical systems monitored by the RXTE-ASM, which cover a spectral band of x-ray photons corresponding to 1.5–12 keV [13], and from Refs. [14,15], we have identified 27 black hole candidates (BH), 48 low mass systems that possess neutron stars as compact objects (LM) and 34 high mass ones (HM). We take the one-day time series of x-ray intensities of each one of these 155 XRBS and transform them into histograms of x-ray intensity I (photon counts/second) variations, $\Delta I = I - \langle I \rangle$. The height of each x-ray intensity value in the histogram is directly proportional to the time-independent probability of occurrence of such an intensity. We also scaled the histograms to 1 since we are not interested in the absolute value of the amplitudes. Each one of the individual histograms of x-ray intensity variations for all 140 sources is well fitted, as the inset in Fig. 1 shows, by a q-Gaussian distribution given by

$$P_q\,(x \equiv I; x_0 \equiv \langle I \rangle) \equiv e_q^{-B_q(x-x_0)^2}. \tag{3}$$

This q-Gaussian arises from the maximization of the q entropy [Eq. (1)] and the parameter B_q as a measure of the dispersion of the distribution. Fifteen histograms were not fitted by the q-Gaussian; in these fifteen histograms eight (gx339−4, ks1731−260, grs1915+105, cygx1, cygx3, gx17+2, cirx1, gx349+2) have clear bimodal distributions and are well fitted by two q-Gaussian distributions (data not shown).

An XRBS consists of compact objects, either in a black hole or a neutron star (accretor) accreting mass from a normal companion star (donor). A low mass x-ray binary (LM) is a binary star where one of the components is either a black hole or a neutron star. The other, donor, component usually fills its Roche lobe and therefore transfers mass to the compact star. The donor is less massive than the compact object, and can be on the main sequence, a degenerate dwarf (white dwarf), or an evolved star (red giant). In this case x-rays are seen only if the two stars are very close to each other, so the companion (normal) star fills its critical gravitational potential Roche lobe. We recall that the Roche lobe is the region of space around a star within which orbiting material is gravitationally bound to that star. If the star expands past its Roche lobe, then the material outside of the lobe will fall into the other star. A high mass x-ray binary (HM) is a binary star system that is strong in x-rays, and in which the normal stellar component is a massive star. The compact component is generally a neutron star, a black hole, or possibly a white dwarf. A fraction of the stellar wind of the massive normal star is captured by the compact object, and produces x-rays as it falls onto the compact object. In this case the companion star mass is typically ten times that of our sun. These high mass stars usually have an intense wind of stellar gas, that is easily captured by the neutron star or black hole. The x-ray emission is due to the process of accretion of the gas by the black hole or neutron star. The gas follows a spiral orbit mainly due to the viscosity in the disk, that redistributes the angular momentum among the gas elements, until captured by the black hole or neutron star. This process causes the gas to be heated by the released gravitational energy, and the heating, of the order of several million degrees, is sufficient for gas to form a plasma that radiates x-rays. However, the usual friction due to viscosity is insufficient alone to drive the accretion. Shakura and Sunayev in 1973 proposed turbulence in the gas as the source of an increased friction [16]. However, this turbulence cannot be explained by standard hydrodynamics since the accretion disk is stable to hydrodynamic perturbations and the fluid is expected to be laminar. For a turbulence to exist, some form of nonlinear hydrodynamic instability is necessary. Balbus and Hawley in 1991 showed that the accretion disk becomes unstable in the presence of a weak magnetic field, the so-called magneto-rotational instability [17], a form

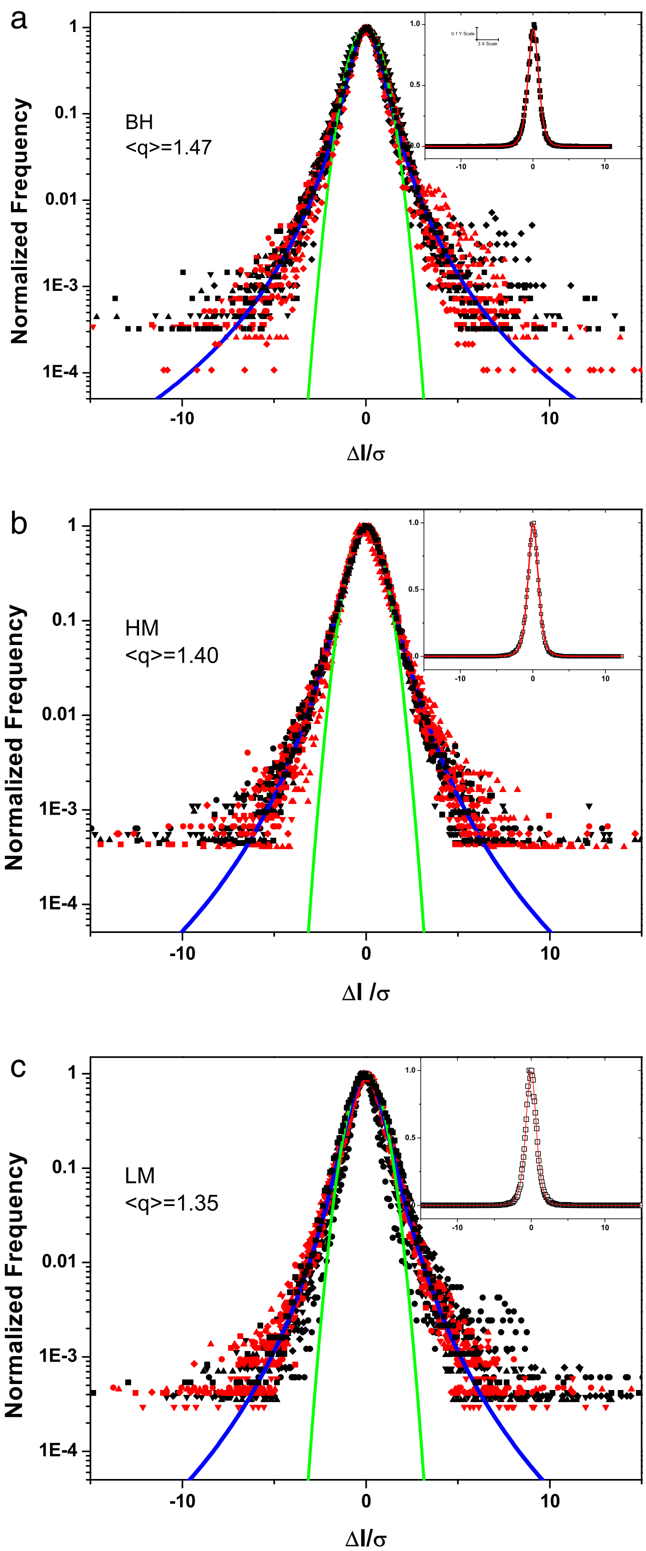

Fig. 1. (Color Online) Normalized histograms of the x-ray light curve intensity variation. The points represent all systems identified (a) as black hole candidates, (b) as high mass neutron stars and (c) as low mass neutron stars. The outermost curve represents a q-Gaussian distribution obtained from the q-averaged value of all the q obtained from the individual curve fittings. The innermost curve indicates a Gaussian fit of these systems. The inset in each subfigure shows a histogram of a individual system with the corresponding q-Gaussian fit curve (on a linear scale).

of magnetohydrodynamic turbulence. The magnetohydrodynamics is subtly different from the hydrodynamics since the tension forces of a magnetic field have no correspondence in the usual hydrodynamic regime. The energy associated with the magnetic field depends only on the magnetic moment of the dipole of the compact object and on the distance from it considered. The x-ray intensity variations registered in the time series, have, as discussed, a direct relationship with how a mass changes as a function of the time [18].

The process thus described is better analyzed by means of the Fokker–Planck equation (FPE). In our case, the FPE gives the time-independent x-ray intensity probability, i.e. $P_q(x \equiv I)$. Still, due to nonlinear magneto-rotational instabilities, the standard linear FPE is not appropriate. A nonlinear FPE is needed, whose drift term includes a nonlinear response accounting for the magnetohydrodynamic instabilities originated in the gas orbit. The FPE solution must be the probability distribution that maximizes the Tsallis entropy, i.e. the q-Gaussian distribution Eq. (3). The stationary FPE that fulfills the above requirements is

$$D\frac{\mathrm{d}P_q(x)}{\mathrm{d}x} + K_{nl}(x)P_q(x) = 0, \tag{4}$$

where D is the diffusion coefficient and $K_{nl}(x) = -\alpha(x-x_0)P_q(x)^{q-1}$ is the nonlinear drift coefficient, with both D and K taken in the I-space. Eq. (4) is a generalization of the FPE with a linear Ornstein–Uhlenbeck process [19] and linear drift coefficient, $K_l(x) = -\alpha(x - x_0)$. Notice that the nonlinear drift term accounts for the fluctuations produced in the temperature of the gas in the accretion disk caused by turbulence due to nonlinear magnetohydrodynamic instability. The turbulence strength is necessarily related to $P_q(x)^{q-1}$. Hence, q is associated with the degree of turbulence in the gas spiral flow before accretion. The diffusion term with a constant diffusion coefficient accounts for a homogeneous temperature inside the accretion disk. Eq. (4) fits the histograms of each one of the x-ray intensity variations of XRBS (except the bimodal distribution) as shown in Fig. 1. These solutions are natural generalizations of the Gaussian solutions for the linear Ornstein–Uhlenbeck process [19]. The GTS provides one proper way to study these nonextensive and incomplete equilibrium XRBS. Note that the Renyi entropy also leads to power-law distributions with long tails. However, the Renyi entropy is extensive and, in addition, only satisfies the concavity criterion (relevant for the thermodynamical stability of the system) for the interval $0 < q \leq 1$. It also violates the property of finiteness of the entropy production per unit time when $q \neq 1$ [20]. It seems inappropriate for XRBS. The galaxy distribution is, as an example, better fitted by the Tsallis distribution than by Renyi distribution [21].

For all the 140 single-peak x-ray light curves analyzed, more than 96% have $q > 1$ in accordance with theoretical prediction [22–24]. On the other hand, the hydrostatic equilibrium is a fundamental assumption of SGS. At hydrostatic equilibrium, there is a balance between the thermal pressure and the weight of the material pressing inwards. In a recent work, Du shows that hydrostatic equilibrium occurs in a SGS only if $q < 1.4$ [25]. Thus, we obtained the normalized histogram (Fig. 1) on the basis of the x-ray light intensity variations for all BH Fig. 1(a), HM Fig. 1(b) and LM Fig. 1(c). For each system we obtained, via a q-Gaussian fit, the corresponding q value. The values of q for these XRBS are in the vicinity of the hydrostatic equilibrium. After this procedure we calculate the average value of q, i.e., for BH this value is $\langle q\rangle = 1.47 \pm 0.13$, for HM it is $\langle q\rangle = 1.40 \pm 0.11$ and for LM it is $\langle q\rangle = 1.35 \pm 0.24$. Also in Fig. 1 we present (for visual effect) the q-Gaussian (outermost curve) taking into account the $\langle q\rangle$ and we show the difference between this mean q-Gaussian and a Gaussian fit.

If we analyze the results by using the non-parametric Mann–Whitney test [26], the BH show a great tendency to break the hydrostatic equilibrium ($q = 1.4$) of the x-ray binary systems. From the results obtained using the non-parametric Mann–Whitney test we reject, with $\alpha = 0.01$ as confidence level, the hypothesis that the entropic indexes for x-ray binary systems with a black hole as the compact object are the same as the entropic indexes for x-ray binary systems with neutron stars as the compact objects.

In summary, we study histograms of the x-ray intensity variations obtained from the time series of 155 light curves of x-ray binary systems. The intensity distributions are well fitted by q-Gaussian distributions which in turn satisfy a nonlinear Fokker–Planck equation. Since the entropic index q is known to reflect fractality [27], the present result strongly indicates that the generation of x-rays occurs in scale invariant media, as observed in Ref. [14,28]. In this sense, the light curves show a clear and universal self-affine behavior [28] that is only departed from when a flare occurs in an x-ray binary system.

The q-Gaussian distribution points to a strong nonextensive and nonequilibrium behavior of these extreme SGS. We show that these gravitating systems can be consistently interpreted in terms of GTS, forging a two-way link between theory and experiment. Conforming with the Mann–Whitney test the binary systems containing black holes have a tendency to be the more nonextensive, with the hydrostatic equilibrium condition being usually broken.

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