



## Cross-correlation between time series of vehicles and passengers

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### ABSTRACT

We study in this paper a cross-correlation between time series of vehicles and passengers collected in the ferry-boat system (sea route that connects the city of Salvador and Itaparica island, Bahia, Brazil), this study is based on the detrended cross-correlation analysis (DCCA) method. The DCCA method is designed to investigate power-law cross correlations between different simultaneously recorded time series in the presence of nonstationarity. Here in this paper we show that is possible to discriminate cross-correlation between vehicles and passengers and also identify seasonal components.

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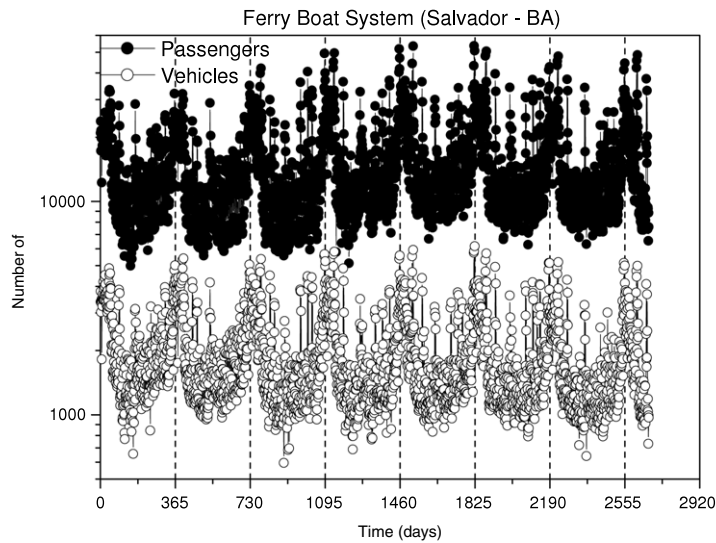
### 1. Introduction

Ship transportation is the process of moving people, vehicles, manufactured products by boat, sailboat, ferry-boat, amongst others, over a sea, lake, canal or river [1–7]. This is frequently undertaken for the purposes of commerce, recreation or military objectives. Although relatively slow, modern sea transport is a highly effective method of transportation. A ferry-boat is a ship carrying passengers, and possibly their vehicles, on a relatively short-distance, regularly-scheduled service. Ferry-boats form an important part of the public transport systems of many waterside cities, allowing direct transit between points on the mainland and islands at a lower cost than bridges or tunnels. In this context the city of Salvador Bahia (Brazil) offers a wide range of opportunities in ship transportation, because Salvador sits on a vast bay, with 1100 km<sup>2</sup>, 70 km from north to south, and 60 km from east to west (at its widest point) and is the largest bay in Brazil. If we look from Salvador for the other side of the bay, we see the Itaparica island. A most common way to travel from Salvador to Itaparica is via the ferry-boat system [8].

In this paper we study cross-correlations between two time series, i.e., vehicles and passengers, collected in the ferry-boat system. This ferry-boat connecting Salvador to Itaparica, has historical and economic importance and requires great logistics for its accomplishment. The ferry-boat system transports vehicles and people from the continent to the island and a great number of tourists mainly in the days of summer. Considering the fixed and floating population, and the area of influence (composed by 30 cities) it is estimated that 5 million users per year use the ferry-boat system. This fact leads this complex system to obey seasonal effects and climatic behavior. As a general rule, the daily demand of the summer is greater than the daily demand of the winter (seasonal component). The seasonal component can be minimized if we apply new statistics methods like the detrended fluctuation analysis (DFA) [9–11] and if we apply new methods, like detrended cross-correlation analysis (DCCA) [12].

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**Fig. 1.** The original time series of passengers (●) and vehicles (○). These data correspond to the demand of the ferry-boat system collected between Jan. 1, 1996 to Apr. 26, 2003 daily. The vertical lines correspond to annual intervals.

## 2. Discussion

Recently Filho et al. [13] show the behavior of the  $\alpha$  exponent (DFA method) and the density of crossing two mobile averages  $\rho$  [14], year by year, in a time series of vehicles. In this paper we use the time series of vehicles (○) and also passengers (●) by using the ferry-boat system, collected daily between Jan. 1, 1996 and Apr. 26, 2003 (see Fig. 1). We are interested in investigating the power-law cross-correlation between these time series and identify seasonality effects by DFA and mainly DCCA methods. The DFA method [10] was proposed to analyze long-range power-law correlations in nonstationary systems and provide a relationship between  $F_{\text{DFA}}(n)$  (root mean square fluctuation) and the box size  $n$ , characterized for a power-law  $F_{\text{DFA}}(n) \propto n^\alpha$ . In this way,  $\alpha$  is the scaling exponent, a self-affinity parameter representing the long-range power-law correlation properties of the signal (fractal properties), such that if  $\alpha = 0.50$  the signal is uncorrelated, if  $\alpha < 0.50$  the correlation in the signal is antipersistent, and if  $\alpha > 0.50$  the correlation in the signal is persistent. Several applications have been made via DFA [15–23].

Now we present the detrended cross-correlation method (DCCA) [12]. DCCA is a generalization of the DFA method and is based on a detrended covariance. This method is designed to investigate power-law cross correlations between different simultaneously recorded time series in the presence of nonstationarity. These consider two long-range cross-correlated time series  $\{y_i\}$  and  $\{y'_i\}$  of equal length  $N$ , compute two integrated signals  $R_k \equiv \sum_{i=1}^k y_i$  and  $R'_k \equiv \sum_{i=1}^k y'_i$ , where  $k = 1, \dots, N$ . After we divide the entire time series into  $N - n$  overlapping boxes, each containing  $n + 1$  values. For both time series, in each box that starts at  $i$  and ends at  $i + n$ , we define the local trend,  $R_{k,i}$  and  $R'_{k,i}$  ( $i \leq k \leq i + n$ ), to be the ordinate of a linear least-squares fit. We define the detrended walk as the difference between the original walk and the local trend. Next we calculate the covariance of the residuals in each box  $f_{\text{DCCA}}^2(n, i) \equiv 1/(n + 1) \sum_{k=i}^{i+n} (R_k - \tilde{R}_{k,i})(R'_k - \tilde{R}'_{k,i})$ . Finally, we calculate the detrended covariance by summing over all overlapping  $N - n$  boxes of size  $n$ ,

$$F_{\text{DCCA}}^2(n) \equiv (N - n)^{-1} \sum_{i=1}^{N-n} f_{\text{DCCA}}^2(n, i). \quad (1)$$

When only one random walk is analyzed ( $R_k = R'_k$ ), the detrended covariance  $F_{\text{DCCA}}^2(n)$  reduces to the detrended variance  $F_{\text{DFA}}^2(n)$  used in the DFA method. If a self-similarity appears, then  $F_{\text{DCCA}} \sim n^\lambda$ . With a brief description of the DFA and the DCCA, then we present the results below.

## 3. Results

Fig. 2 shows the log–log plot of  $F_{\text{DFA}}$  in a function of  $n$  for vehicles (○) and passengers (●) that travel in the ferry-boat system. In this figure we can see appear clearly a persistent/anti-persistent transition, as well as we suspect that there is a strong cross-correlation between these time series (vehicles and passengers). In order to verify this possible cross-correlation we apply the DCCA method in these time series of vehicles and passengers (Fig. 3). The data distribution shows categorically that there is a cross-correlation between vehicles and passengers. If we look at Fig. 3, as well as Fig. 2, we can see the persistent/anti-persistent transition in  $n = 365$ . This value show clearly the influence of annual seasonality (summer and carnival period).

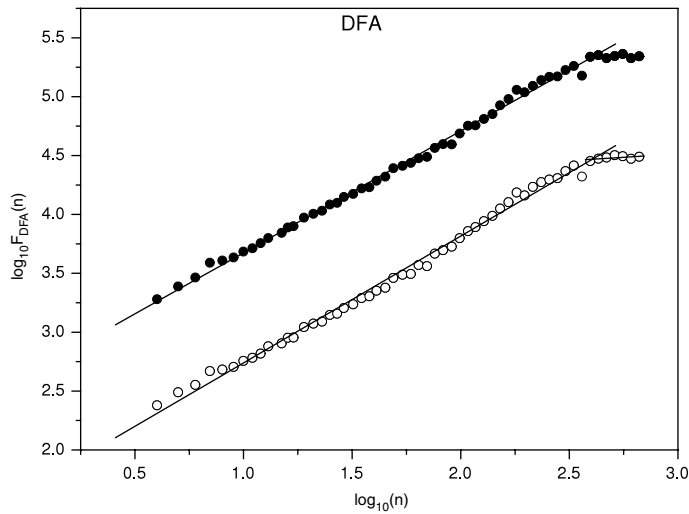


Fig. 2. The DFA method applied in the time series of the ferry-boat system for passengers (●) and vehicles (○).

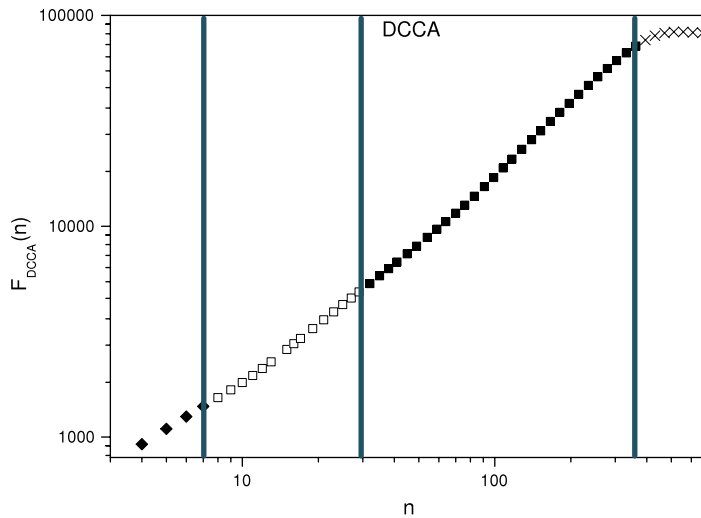


Fig. 3. Detrended cross-correlation analysis (DCCA) between passengers and vehicles. The vertical lines show the intervals ( $n = 7, 30$  and  $365$  days).

Table 1

The values of the  $\alpha$  DFA exponent method for passengers (●) and vehicles (○) and also the  $\lambda_{DCCA}$  exponent measured in the intervals  $n = 4-7, n = 8-30, n = 31-365$  and finally  $n > 365$  days. The (#) represents the standard deviation. The five columns represent the  $\lambda_{DCCA}$  for the absolute values of the successive differences between passengers and vehicles.

Days	$\alpha_{DFA}$ (●)	$\alpha_{DFA}$ (○)	$\lambda_{DCCA}$	$\lambda_{DCCA}$ (abs)
7	1.23 (0.13)	1.14 (0.12)	0.74 (0.01)	0.58 (0.02)
30	0.98 (0.02)	0.96 (0.02)	0.91 (0.01)	0.83 (0.01)
365	1.08 (0.02)	1.17 (0.02)	1.10 (0.01)	0.65 (0.02)
>365	0.00 (0.07)	0.11 (0.07)	0.13 (0.05)	0.20 (0.05)

In order to characterize this transition and to investigate the seasonality effects we propose in this paper to split the DCCA results into intervals in which the seasonal component is possibly present: the (a) interval between  $n = 4$  and  $n = 7$  (weekly), (b) interval between  $n = 8$  and  $n = 30$  (monthly), (c) interval between  $n = 31$  and  $n = 365$  (annual) and (d) finally  $n > 365$ . These intervals are represented by vertical lines in the Fig. 3. For each interval we calculated the  $\lambda$  exponent (that characterizes the long-range cross-correlation, by the DCCA method) and the  $\alpha$  exponent (DFA method) too. All values of  $\lambda_{DCCA}$  and  $\alpha_{DFA}$  exponents are listed in Table 1. Table 1 gives us a complete view of the seasonality effect in the ferry-boat system, because it was possible to measure the persistent/anti-persistent transition and see small seasonal components.

In conclusion,  $\alpha$  and  $\lambda$  exponents (Table 1) show an interesting behavior. For example, if  $n \sim 7$  the time series is persistent with  $\lambda = 0.74$ , in the range of  $n = 30$  the value of  $\lambda$  is 0.91 (close to  $\lambda = 1.0$ ). In the case of  $n \sim 365$ , we can see the persistent

( $\lambda = 1.10$ )/anti-persistent ( $\lambda = 0.13$ ) transition, this transition is seen clearly in Fig. 3 and also (not so clearly) in Fig. 1. For curiosity, we implemented in Table 1, column 5, the value of  $\lambda$  in the case of the magnitude time series, i.e., we analyze the time series of absolute values of the successive differences of passengers and vehicles (see Ref. [12]). The values of  $\lambda$  in these two cases (column 4 and 5) have the same qualitative comportment. Finally, we can identify via the DCCA method small transitions occurring (more complex to be seen), such as days/weeks and weeks/months. Therefore, a study of cross-correlations by the DCCA method, from the point of view of the  $\lambda$  exponent, can be very useful to identify seasonality in correlated time series as in the case of the ferry-boat system.

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