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DCCA cross-correlation coefficient apply in time series of air temperature and air relative humidity

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ABSTRACT

In this paper we propose, analyze and also quantify cross-correlations between climatological data. For this purpose we adopt the DCCA cross-correlation coefficient ρ_{DCCA}. In order to accomplish this goal, we calculate the cross-correlation between time series of air temperature and relative humidity. This analysis was performed taking into account several stations (cities) around the world. The results found here, depending on the station location, may exhibit one of the following behaviors, i.e., negative, positive, or null cross-correlations. It is noteworthy that, the level of cross-correlation between air temperature and relative humidity is quantified in these cases. Finally, DCCA cross-correlation coefficients show that, in general, the data are influenced by seasonal components.

1. Introduction

Global warming is a concern of scientists around the world. Most of it is caused by human activities (anthropogenic causes), as well as, by natural phenomena like El Niño and La Niña. The components of the climate are never in equilibrium and are constantly varying. Therefore, for this complex system, any change in the components may result in a considerable climate variation. Global warming can be one explanation for the trend of increasing natural disasters in recent years [1]. Therefore, the study of climate can help us in preventing these natural disasters, which usually cause a large number of deaths and a great economic loss. From the perspective on the climatology, mathematical models are important tools. These models are applied for a variety of purposes, like the study of the dynamics of the weather, projections of the future climate, changes in the air temperature, among others. The study of contemporary climates incorporates meteorological data accumulated over many years, such as records of rainfall, temperature, and atmospheric composition [2–4]. Thus, we must properly define the fundamental variables in the sense to study this complex system. To accomplish this goal the World Meteorological Organization (WMO) has defined, in Chapter 5 of Ref. [5], the main climatological surface elements, such as temperature, pressure, wind direction and speed, relative humidity, and others. Progress in weather forecasting and in climate modeling has been significant in recent years [6]. According to Ref. [7], Numerical Weather Prediction (NWP) has been the key to this success, because it uses the power of computers. Most of these models use systems of differential equations based on the laws of physics, fluid motion, and chemistry, and use a coordinate system which divides the planet into a 3D grid. Winds, heat transfer, solar radiation, relative humidity, and surface hydrology are calculated within each grid cell, and the interactions with the neighboring cells are used to calculate the atmospheric properties in the future.

Considering the number of fundamental variables and a large possibility of their applications, in this paper we restrict to identify and quantify cross-correlation between air temperature and relative humidity. For this purpose we take the database

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of different cities around the planet, defined by geographic coordinates (latitude, longitude, and elevation). Air temperature and relative humidity are variables that are still widely studied [8–12], but not from the point of view of the DCCA cross-correlation exponent ρ_{DCCA} [13]. It is noteworthy that, ρ_{DCCA} is defined in terms of the DFA and the DCCA method [14,15], briefly describe in the next section.

2. Discussion

In time series analysis there are some well-known paths to follow [16–18], thus if the time series exhibit complex behavior such as self-affinity, we can apply new strategies for its analysis [19–22]. By this point of view, one of the most frequently cited method to analyze time series of complex problems is the detrended fluctuation analysis (DFA) [14]. This method provides a relationship between $F_{DFA}(n)$ (root mean square fluctuation) and the scale n. DFA method has been very efficient at detecting long-range auto-correlations embedded in a patch landscape and also avoiding spurious detection of apparent long-range auto-correlations. This fact can be proved by a great number of applications and citations [23–33]. However, if we have two time series, $\{y_i\}$ and $\{y'_i\}$, the analysis of cross-correlation can be applied, like in Refs. [34–42]. In this paper we are proposing analyze and quantify cross-correlation for climatological data, specifically between time series of air temperature and relative humidity (daily average values). We adopt the recently proposition implemented by Zebende [13], based on the Detrended Cross-Correlation Analysis method (DCCA) [15].

The DCCA method is a generalization of the DFA method and is based on detrended covariance. This method is designed to investigate power-law cross-correlations between different simultaneously recorded time series in the presence of nonstationarity. Therefore, for two time series of equal length N, we compute two integrated signals $R_k \equiv \sum_{i=1}^{k} y_i$ and $R'_k \equiv \sum_{i=1}^{k} y'_i$, where $k = 1, \ldots, N$. In the next step we divide the entire time series into $N - n$ overlapping boxes, each containing $n + 1$ values. For both time series, in each box that starts at i and ends at $i + n$, we define the local trend, $\widetilde{R}_{k,i}$ and $\widetilde{R}'_{k,i}$ $(i \leq k \leq i + n)$, to be the ordinate of a linear least-squares fit. We define the detrended walk as the difference between the original walk and the local trend. Next we calculate the covariance of the residuals in each box $f^2_{DCCA}(n, i) \equiv 1/(n+1) \sum_{k=i}^{i+n} (R_k - \widetilde{R}_{k,i})(R'_k - \widetilde{R}'_{k,i})$. Finally, we calculate the detrended covariance function by summing over all overlapping $N - n$ boxes of size n:

$$F^2_{DCCA}(n) \equiv (N - n)^{-1} \sum_{i=1}^{N-n} f^2_{DCCA}(n, i). \tag{1}$$

When only one random walk is analyzed, $(R_k = R'_k)$, the detrended covariance $F^2_{DCCA}(n)$ reduces to the detrended variance $F^2_{DFA}(n)$ used in the DFA method. If self-affinity appears, then $F^2_{DCCA}(n) \sim n^{2\lambda}$. DCCA has been applied in many situations [43–48]. The λ exponent quantifies the long-range power-law cross-correlations and also identifies seasonality [46], but λ does not quantify the level of cross-correlations.

To quantify the level of cross-correlation, we can apply the DCCA cross-correlation coefficient [13], defined as the ratio between the detrended covariance function F^2_{DCCA} and the detrended variance function F_{DFA}, i.e.,

$$\rho_{DCCA} \equiv \frac{F^2_{DCCA}}{F_{DFA\{y_i\}} F_{DFA\{y'_i\}}}. \tag{2}$$

Eq. (2) leads us to a new scale of cross-correlation in nonstationary time series. It is to be noted that in Ref. [13] the Eq. (2) was typed incorrectly. The value of ρ_{DCCA} ranges between $-1 \leq \rho_{DCCA} \leq 1$. A value of $\rho_{DCCA} = 0$ means there is no cross-correlation, and it splits the level of cross-correlation between positive and the negative case (see Table 1 in Ref. [13]). Exponent ρ_{DCCA} has been tested on selected time series, simulated and real cases, and has proved to be quite robust.

3. Data and results

Taking into account the values of the successive differences, Podobnik and Stanley [15] find that there are power-law autocorrelations (by DFA) and power-law cross-correlations (by DCCA) between these time series, and $F^2_{DCCA}(n)$ presents a negative value for every n. However, to present their results (Fig. 2 of Ref. [15]) Podobnik and Stanley considered only the absolute values of the successive differences $|\{y'_{i+1} - y'_i\}|$. In addition to the above results, they find that both time series show sudden bursts of large changes. Nevertheless, these results of cross-correlations analysis were obtained from an only city and do not predicted possible changes in terms of location (latitude, longitude, and elevation).

In order to amplify the study of cross-correlation between air temperature and relative humidity, we propose here measure the value of ρ_{DCCA} as a function of n (the time scale) for more cities. In this paper we analyze the successive differences of relative humidity $\{y_{i+1} - y_i\}$ and air temperature $\{y'_{i+1} - y'_i\}$, differently than was proposed in Ref. [15, Fig. 2], i.e., the absolute values of the successive differences. This choice was made simply because we can see directly whether the time series are anti cross-correlated. If we analyze absolute values of the successive differences, only positive (or null) values of cross-correlation can appear (see Fig. A.1 (□)). Thus, here we can see directly what kind of cross-correlation exists between air temperature and relative humidity (positive, negative, or null cross-correlations), as well as quantify such cross-correlations in function of time scale by the DCCA cross-correlation coefficient ρ_{DCCA}. Our data were obtained from the

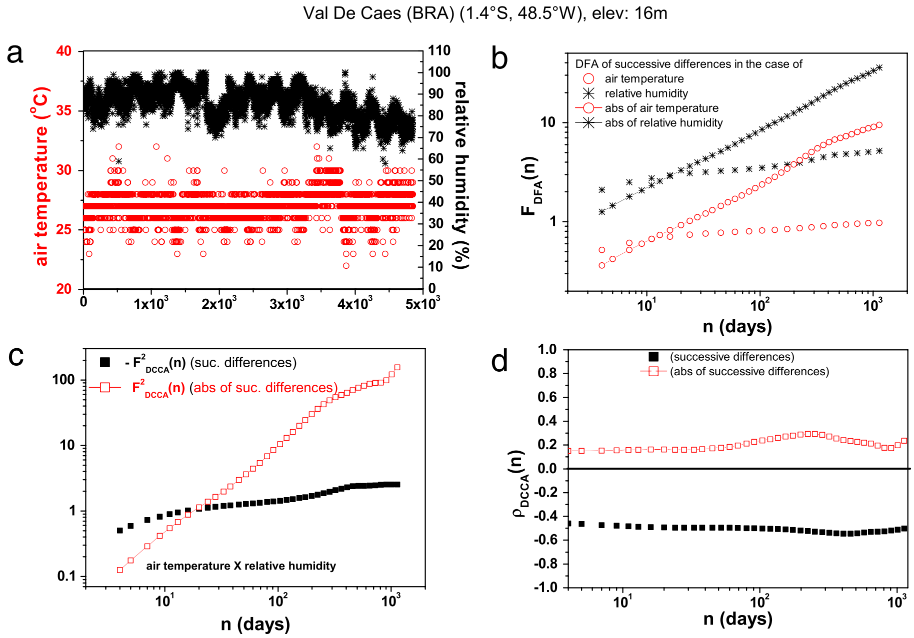

Fig. A.1. (Color on line) DCCA cross-correlations analysis for the time series of air temperature and relative humidity in the case of the city of Val De Caes (BRA) (1.4 S, 48.5 W), elev: 16 m. In this figure we show the original time series for air temperature (red) and relative humidity (black) (a), the DFA analysis for successive differences of the relative humidity (∗) and the air temperature (∘) (b), the DCCA cross-correlation analysis (■) (c), and the value of ρ_{DCCA} in function of n (■) (d). Here we also present (cases b, c, and d), the results for cross-correlation analysis in terms of the absolute values of the successive differences (symbol + line), as was proposed by Podobnik and Stanley [15].

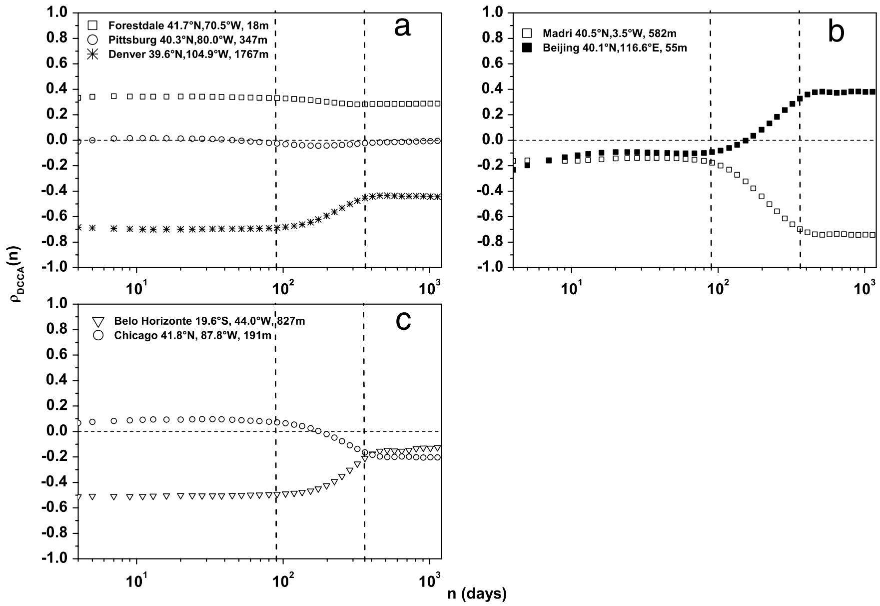

Fig. A.2. DCCA cross-correlation coefficient ρ_{DCCA} between air temperature × relative humidity: (a) corresponds approximately to latitude 40° (USA), (b) corresponds approximately to latitude 40° others countries, and (c) others latitudes. The vertical lines corresponds to 90 and 365 days respectively (in time scale).

Weather Underground database [49], a committee that monitors conditions and forecasts for locations across the world. Therefore, for a given locality, we obtain the daily average values of air temperature and relative humidity, recorded from 01 January 1997 to 30 November 2010. Fig. A.1 shows an example for DCCA cross-correlation calculation. In this figure we can see: (a) the original time series of the air temperature (∘) and the relative humidity (∗), (b) the DFA auto-correlation analysis, (c) the DCCA cross-correlation analysis, and lastly, (d) the value of the exponent ρ_{DCCA}, as a function of n. This figure also present, for comparison with Ref. [15], the analysis take in account the absolute values of the successive differences

Table A.1
Mean values of ρ_{DCCA}, with seasonal components.

Locality	Week	Month	Season	Year	>year
Quito (ECU)	−0.31	−0.32	−0.35	−0.35	−0.33
Nairobi (KEN)	−0.38	−0.35	−0.36	−0.34	−0.33
Medan (IDN)	−0.81	−0.81	−0.80	−0.80	−0.80
Changi (SGP)	−0.62	−0.62	−0.64	−0.64	−0.65
Val De Caes (BRA)	−0.47	−0.49	−0.50	−0.52	−0.53
Libreville (GAB)	−0.46	−0.44	−0.40	−0.30	−0.22
Malé (MDV)	−0.46	−0.44	−0.44	−0.41	−0.40
Bariloche (ARG)	−0.27	−0.26	−0.28	−0.43	−0.66
Puerto Montt (CHL)	−0.01	0.03	0.05	−0.10	−0.36
Wellington (NZL)	−0.02	0.08	0.10	0.05	−0.06
Hobart (TAS) g	−0.25	−0.20	−0.20	−0.26	−0.36
Mexico city (MEX)	−0.40	−0.39	−0.37	−0.29	−0.12
Tegucigalpa (HND)	−0.37	−0.33	−0.31	−0.32	−0.28
San José (CRI)	−0.37	−0.37	−0.40	−0.43	−0.44
Havana (CUB)	−0.13	0.03	0.09	0.15	0.24
Jeddah (SAU)	−0.21	−0.17	−0.13	−0.11	−0.14
Hilo Hawaii (USA)	−0.24	−0.16	−0.17	−0.15	−0.09
Chek Lap Kok (HKG)	−0.06	0.08	0.11	0.15	0.25
Hanoi (VNM)	−0.27	−0.08	0.01	0.05	0.12
Owen Roberts (CYM)	−0.19	−0.09	−0.06	−0.03	0.03
Nouakchott (MRT)	−0.68	−0.67	−0.64	−0.38	0.00
Denver (USA)	−0.69	−0.70	−0.69	−0.58	−0.44
Provo (USA)	−0.46	−0.45	−0.45	−0.59	−0.78
Madrid (ESP)	−0.16	−0.15	−0.15	−0.44	−0.74
Sofia (BGR)	−0.32	−0.28	−0.29	−0.39	−0.54
Beatrice (USA)	−0.15	−0.18	−0.18	−0.21	−0.26
Pittsburgh (USA)	0.00	0.01	−0.01	−0.04	−0.01
Columbus (USA)	0.08	0.10	0.09	0.01	−0.06
Kansas city (USA)	−0.08	−0.09	−0.10	−0.09	−0.07
Chicago (USA)	0.08	0.09	0.09	−0.03	−0.20
St. Louis (USA)	−0.02	0.01	0.01	−0.02	−0.06
Lisboa (PRT)	−0.33	−0.38	−0.37	−0.43	−0.54
Roma (ITA)	−0.03	0.02	0.03	−0.14	−0.38
Belgrade (SRB)	−0.44	−0.43	−0.45	−0.49	−0.57
Akita (JPN)	−0.14	−0.15	−0.17	−0.14	−0.12
Athens (GRC)	−0.17	−0.11	−0.14	−0.37	−0.65
Tirana (ALB)	−0.22	−0.07	0.01	−0.06	−0.17
Arcata (USA)	−0.25	−0.18	−0.14	−0.08	0.03
Beijing (CHN)	−0.20	−0.11	−0.10	0.10	0.38
Istanbul (TUR)	−0.25	−0.19	−0.17	−0.28	−0.47
New York (USA)	0.15	0.14	0.12	0.16	0.26
Algiers (DZA)	−0.41	−0.39	−0.36	−0.39	−0.49
Philadelphia (USA)	0.21	0.22	0.20	0.18	0.18
Forestdale (USA)	0.34	0.34	0.34	0.30	0.29
Tunis-Carthage (TUN)	−0.37	−0.34	−0.33	−0.42	−0.60
Potosi (BOL)	−0.43	−0.43	−0.42	−0.16	0.15
Belo Horizonte (BRA)	−0.51	−0.51	−0.50	−0.39	−0.14
La Tontouta (NCL)	−0.27	−0.13	−0.07	−0.07	−0.07
Beira (MOZ)	−0.60	−0.50	−0.45	−0.39	−0.34
Townsville (AUS)	−0.08	0.08	0.15	0.15	0.22
Taiti (PYF)	−0.18	−0.11	−0.11	−0.06	0.03
Mean values	−0.25	−0.21	−0.20	−0.21	−0.22

(symbol + line, cases (b), (c), and (d)). As previously tested [13], ρ_{DCCA} gives a great summary of the analysis proposed by the DFA and DCCA methods, with the advantage of quantify the level of cross-correlation between these time series.

Now differently from Podobnik and Stanley [15], we can show the behavior of the cross-correlation between air temperature and relative humidity as a function of n, thinking in terms of ρ_{DCCA}, see Fig. A.2. In this figure we can see different types of cross-correlations depending on the city localization. In order to exemplify this statement (a more complete study will be presented after, in Table A.1), we divide Fig. A.2 into three parts, which are: Fig. A.2(a), same latitude and same country, clearly with the three types of cross-correlation, positive (□), negative (∗), and null case (○); Fig. A.2(b), same latitude and different countries, in this case we can see that for small values of n (the time scale) ρ_{DCCA} have approximately the same negative value for Madrid (□) and Beijing (■), but for large values of n is positive for Beijing and negative for Madrid; Fig. A.2(c), different latitudes and different countries, for small values of n the exponent ρ_{DCCA} is positive for Chicago (○) and is negative for Belo Horizonte (▽), while for large values of n, ρ_{DCCA} tends to a same negative value.

In general, apart from these three types of cross-correlation registered in Fig. A.2, we identify seasonal components (vertical lines in Fig. A.2). These seasonal components are much more complicated to be identified if we look only the value of $F_{DCCA}(n)$. For example, in Fig. A.2, we can identify a clear pattern at $n = 365$ (annual component), but depending on the specific city, other types of seasonal components can arise, like in $n = 90$. In order to have a more complete view, we did the analysis taking into account 51 cities, with distinct latitudes, longitudes, and elevations (see Appendix). Certainly, we can identify all types of cross-correlation behavior, and identification of seasonality, in function of time scale.

4. Conclusions

Podobnik and Stanley in Ref. [15] found that DCCA analysis has F^2_{DCCA} negative for every n, for values of the successive differences of air temperature and relative humidity. This assertion featuring an anti cross-correlation behavior between air temperature and relative humidity for a given location. However, we cannot claim that the cross-correlations between air temperature and relative humidity are negative for a specific place in the earth. In order to justify such proposition, we obtain all types of cross-correlations between successive differences of air temperature and relative humidity, and quantified here by ρ_{DCCA}. For example, the value of ρ_{DCCA} ranges from 0.34 (Forestdale, USA) to -0.81 (Medan, IDN). We should also enhance here that, by the DCCA cross-correlation exponent, we can identify directly seasonal components, like presented in Fig. A.2, vertical lines at $n = 90$ and $n = 365$ respectively. As a final test, as a curiosity, we calculate the arithmetic mean of the ρ_{DCCA} in function of n, for all 51 cities (last line in Table A.1). In this case we find a anti cross-correlation trend with $\rho_{DCCA} \simeq -0.22$.

Finally, we propose the application of the ρ_{DCCA} for cross-correlation analysis in climatological data, specifically in time series of air temperature and relative humidity. Here, we introduce new cities and we show that, depending on the city location, the cross-correlation can be negative, positive or null. We have noticed that it is the first time that the value of cross-correlation between air temperature and relative humidity is quantified, by way of ρ_{DCCA}. Logically, this study can be extended to treat other types of climatological data, due the generality of ρ_{DCCA}.

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Appendix. Complete list of the cities

Below (Table A.1), we present the complete study in terms of the exponent ρ_{DCCA} for 51 cities around the world. The values contained in this table represents the mean values of ρ_{DCCA} taking the intervals:

- 4 and 7, week seasonality;
- 8 and 30, month seasonality;
- 31 and 90, season seasonality;
- 91 and 365, year seasonality;
- $>$365, long range seasonality.

In the last line of Table A.1, we print the mean value (between columns) for all 51 cities. These mean values have $\rho_{DCCA} \simeq -0.21$. This value, for the global case, show a anti cross-correlated behavior.

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